Quick Study® ACADEMIC

PRE-ALGEBRA

NUMBER SYSTEMS

The following sets are infinite; that is, there are no last numbers. The three dots indicate continuing or never-ending patterns.

- **Counting or natural numbers** = {**1, 2, 3, 4, ..., 78, 79, ...**}.
- **Whole numbers** = {**0, 1, 2, 3, 4, ..., 296, 297, ...**}.
- **Integers** = {**..., –4, –3, –2, –1, 0, 1, 2, 3, 4...**}.
- **Rational numbers** = {all numbers that can be written as fractions, p/q, where p and q are integers and q is not zero}. Rational numbers include all counting numbers, whole numbers and integers, in addition to all proper and improper fraction numbers, and ending or repeating decimal numbers.
  Ex: $4/9$, $.\overline{3}$, -18.75, $\sqrt{16}$, $-\sqrt{25}$.
- **Irrational numbers** = {all numbers that cannot be expressed as rational numbers}. As decimal numbers, irrational numbers do not end or repeat.
  Ex: $3.171171117\ldots$, $\sqrt{7}$, $-\sqrt{2}$, π.
- **Real numbers** = {all rational and all irrational numbers}.

OPERATIONS

ABSOLUTE VALUE

Absolute value is the distance (**always positive**) between a number and zero on the number line; the positive value of a number. **Ex:** $|3| = 3$; $|-3| = 3$; $|-.5| = .5$.

ADDITION

1. **Integers:** When adding integers, follow these rules.
   a. If both numbers are positive, add them; the sign of the answer will be positive.
   b. If both numbers are negative, add them; the sign of the answer will be negative.
   c. If one number is negative and the other is positive (in any order), subtract the two numbers (even though they are joined by a plus sign); the sign of the answer will be the same sign as the sign of the number that has the larger absolute value.
   Ex: $4 + (-9) = -5$; $(-32) + (-2) = -34$; $(-12) + 14 = 2$.
2. **Rational numbers:**
   a. When adding **two mixed numbers**, **fractions**, or **decimal numbers**, follow the same sign rules that are used for integers (above), but also follow the rules of operations for each type of number.
   b. For **mixed numbers and fractions**, make sure the fractions have a common denominator, then add the numbers. Mixed numbers and fractions can also be changed to decimal numbers and then added.
   c. For **decimal numbers**, line the decimal points up, then add the numbers, bringing the decimal point straight down.
   Ex: $(-4\,^1/_2) + (5\,^3/_4) = (-4\,^2/_4) + (5\,^3/_4) = 1\,^1/_4$; $5.667 + (-.877) = 4.79$.
3. **Irrational numbers:**
   a. When adding irrational numbers, **exact decimal values cannot be used.** If decimal values are used, then they are rounded and the answer is only an approximation. Instead, if the two irrational numbers are **multiples of the same square root, radical expression,** or **pi** (π), then simply add the coefficients (numbers in front) of the roots or **pi** (π).
   Ex: $4\sqrt{3}+5\sqrt{3}=9\sqrt{3}$; $(-6\pi)+9\pi=3\pi$; $3\sqrt{7}+3\sqrt{2}$ cannot be added any further because the two square roots are different.

SUBTRACTION

1. Subtraction of all categories of numbers can be accomplished by adding the opposite of the number to be subtracted.
2. After changing the sign of the number in back of the minus sign, follow the rules of addition as stated above.
   Ex: $8 - (-3) = 8 + (+3) = 11$; $(-15) - (9) = (-15) + (-9) = -24$.

MULTIPLICATION

1. **Integers:** When multiplying integers, follow these rules.
   a. If the signs of the numbers are the **same**, multiply and make the answer **positive**.
   b. If the signs of the numbers are **different**, multiply and make the answer **negative**.
   c. **NOTE:** The sign of the answer does not come from the number with the larger absolute value as it does in addition.
   Ex: $(-4)(5) = -20$; $(-3)(-2) = 6$; $(7)(-10) = -70$.
2. **Rational numbers:**
   a. When multiplying **rational numbers**, follow the sign rules that are used for multiplying integers (above) and the rules for multiplying each type of number.
   b. For **mixed numbers**, change each mixed number to an improper fraction, and then multiply the resulting fractions.
   c. For **fractions**, multiply the numerators and the denominators, then reduce the answer.
   d. For **decimal** numbers, multiply them as though they were integers, then put the decimal point in the answer so there is the same number of digits behind the decimal point in the answer as there are behind both decimal points in the problem.
3. **Irrational numbers:**
   a. When multiplying irrational numbers, follow the same sign rules that are used for integers (listed above).
   b. If radical expressions are multiplied and they have the same indices, then the numbers (radicands) under the root symbols (radicals) can be multiplied.
   Ex: $(-\sqrt{5})(\sqrt{7}) = -\sqrt{35}$; $(3\sqrt{7})(-4) = -12\sqrt{7}$.

DIVISION

1. **Integers:** When dividing integers, follow these rules:
   a. If the signs of the numbers are the **same**, divide them and make the answer **positive**.
   b. If the signs of the numbers are **different**, divide them and make the answer **negative**.
   c. The sign of the answer does not come from the number with the larger absolute value as it does in addition.
   Ex: $(-30)/(5) = -6$; $(-22)/(-2) = 11$; $(70)/(-10) = -7$.
2. **Rational numbers:**
   a. When dividing **rational numbers**, follow the sign rules that are used for dividing integers (listed above) and the rules for dividing each type of number.
   b. For **mixed numbers**, change each mixed number to an improper fraction, invert or flip the number behind the division sign and follow the rules for multiplying fractions.
   c. For **decimal numbers**, first move the decimal point in the divisor to the back of the number, then move the decimal point the same number of positions to the right in the dividend. Divide the numbers, then bring the decimal point straight up into the quotient (answer). Additional zeros can be written after the last digit behind the decimal point in the dividend so the division process can continue if needed.
3. **Irrational numbers:**
   a. When dividing irrational numbers, follow the same sign rules that are used for dividing integers (listed above).
   b. If radical expressions are divided and they have the same indices, then the numbers (radicands) under the root symbols (radicals) can be divided.
   Ex: $(-\sqrt{15})/(\sqrt{3}) = -\sqrt{5}$; $(\sqrt{30})/(-\sqrt{6}) = -\sqrt{5}$; $\sqrt[3]{4}/\sqrt{2}$ cannot be divided, only simplified as demonstrated in the *QuickStudy® Algebra Part One* study guide.

EXPONENTS/POWERS

1. **Definition:** $a^n = \underbrace{a \cdot a \cdot a \cdot a \cdot a \cdots}_{n}$, that is, the number written in the upper right-hand corner is called the exponent or power, and it tells how many times the other number (called the base) is multiplied times itself. If an exponent cannot be seen, it equals **1**. **Ex:** $5^6 = 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 = 15{,}625$; notice that the base, **5**, was multiplied times itself **6** times because the exponent was **6**.
2. **Rule:** $a^n \bullet a^m = a^{m+n}$; that is, when multiplying the same base, the new exponent can be found quickly by adding the exponents of the bases that are multiplied.
   Ex: $(5^3)(5^4) = 5^7$; $(3^2)(7^3)(7^2)(3^5) = (3^7)(7^5)$.
3. **Rule:** $a^n/a^m = a^{n-m}$; that is, when dividing the same base, the new exponent can be found quickly by subtracting the exponents of the bases that are divided. The new base and exponent go either in the **numerator** or in the **denominator**, wherever the **highest exponent** was located in the original problem.
   Ex: $(7^5)/(7^2) = 7^3$; $(3^4)/(3^6) = 1/(3^2)$.
4. **Rule:** $a^{-1} = 1/a$; and $1/a^{-1} = a$; that is, a negative exponent can be changed to a positive exponent by moving the base to the other section of the fraction; numerator goes to denominator or denominator goes to numerator.
   Ex: $7^{-3} = 1/(7^3)$; $1/(5^{-2}) = 5^2$; $3(2^{-4}) = 3/(2^4)$; notice the **3** stayed in the numerator because the invisible exponent is always positive **1**.
5. **Rule:** $(a^n)^m = a^{nm}$; that is, when there is a base with an exponent raised to another exponent, then the short cut is to multiply the exponents.
   Ex: $(-3z^2)^3 = (-3)^3(z^2)^3 = -27z^6$.

ORDER OF OPERATIONS

When a problem has many operations, the order in which the operations are completed will give different answers; so, there is an order of operations rules.

1. Do the operations in the **parentheses** (or any enclosure symbols) first.
2. Do any **exponents or powers** next.
3. Do any **multiplication and division**, going left to right in the order they appear (this means division is done before multiplication if it comes first in the problem).
4. Do the **addition and subtraction**, going left to right in the order they appear (this means subtraction is done before addition if it comes first in the problem).
   Ex: $4 + 2(3 + 7) = 4 + 2(10) = 4 + 20 = 24$; $40 \div 5 \bullet 2 + 4 \div 4 = 8 \bullet 2 + 1 = 16 + 1 = 17$.

SCIENTIFIC NOTATION

A form of a decimal number where the decimal point is always behind exactly one non-zero digit and the number is multiplied by a power of ten.

Ex: 4.87×10^8; 3.981×10^{-6}.

1. It is a method for representing **very large** or **very small** numbers without writing a lot of digits. **Ex: 243,700,000,000,000** would be written as **2.437×10^{14}**; **.000000982** would be written as **9.82×10^{-7}**.
2. A **positive** or **zero** exponent on the **10** means the number value is more than or equal to one. A **negative** exponent on the **10** means the number value is less than one. **Ex: 5.29×10^{-10} = .000000000529** and **5.29×10^{14} = 529,000,000,000,000**.
3. Operations with very large or very small numbers can be completed using the scientific notation form of the numbers, especially with calculators.

ALGEBRA CONCEPTS

DEFINITIONS

1. A **variable** is a letter that represents a number.
2. A **coefficient** is a number that is multiplied by the variable. It is found in front of a variable, but the multiplication sign is not written. If the coefficient is one, the one is not written. **Ex:** $5n = 5 \times n$; if n were **3**, then $5n$ would equal 5×3 or **15**.
3. A **term** is a mathematical expression involving multiplication or division. Terms are separated by an addition or subtraction sign. **Ex:** $7a$ is one term; $3k + 9$ is two terms; $4m^2 - 8m + 3$ is three terms.
4. **Like (or similar) terms** are terms that have the same variables and exponents, written in any order. The coefficients (numbers in front) **do not** have to be the same. **Ex:** $4m$ and $9m$ are like terms; $5a^2c$ and $-7a^2c$ are like terms; $3r^3$ and $-9r^2$ are not like terms because the exponents are not the same; $15z^4$ and $8t^4$ are not like terms because they do not have the same variables.

OPERATIONS & PROPERTIES

1. **Addition and Subtraction:** Only **like** (or similar) terms can be added or subtracted. Once it is determined that the terms are like terms, only the coefficients (numbers in front of the terms) are added or subtracted. **Ex:** $3n + 7n - 11n = 10n - 11n = 10n + (-11n) = -1n$ or simply $-n$; $14k^2 + 5n - 10k^2 - n = 4k^2 + 4n$.
2. **Multiplication: Any terms** can be multiplied. They **do not** have to be like terms. When multiplying terms, multiply the coefficients (numbers in front) and the matching variables. **Ex:** $(-3m^2n)(5m^4n) = -15m^6n^2$; remember, when multiplying, make sure the **bases** are the same, then add exponents.
3. **Division**: **Any terms** can be divided. They **do not** have to be like terms. Division is usually written in fraction form. When dividing terms, divide or reduce the coefficients (numbers in front) and the matching variables. Remember that, to divide with exponents, you must subtract the exponents once you match the same bases. **Ex:** $(30a^7c^2)/(-6a^4c^3d^2) = (-5a^3)/(cd^2)$ because **30** divided by **–6** is **–5**, a^7 divided by a^4 is a^3, c^3 divided by c^2 is c, and there is no other variable d to divide by the d^2, so it remains the same.
4. **Commutative Property**: $a + b = b + a$ and $a - b = a + (-b) = (-b) + a$; therefore, terms can be moved as long as you take the proper sign (negative or positive) with the term. **Ex:** $4p^2 + 8p^3 = 8p^3 + 4p^2$; $14c - 3f = (-3f) + 14c$.
5. **Associative Property**: $(a + b) + c = a + (b + c)$ and $(a - b) - c = a + (-b + -c)$; therefore, terms can be added in any order as long as all subtraction is first changed to addition. **Ex:** $(5j - 8j) - 12j = 5j + (-8j + -12j)$.
6. **Distributive Property**: $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$; therefore, if the terms inside the parentheses cannot be added or subtracted, multiply them BOTH or ALL by the value located in **front** of the parentheses. **Ex:** $3n(5n + 6) = 15n^2 + 18n$; $a^2c(5a^2 + 2ac - c^2) = 5a^4c + 2a^3c^2 - a^2c^3$.
7. **Double Negative Property**: $-(-a) = a$; therefore, if there is a negative of a negative, it becomes positive, just like a negative number times a negative number equals a positive number.

TRANSLATING

PUTTING WORDS INTO ALGEBRAIC STATEMENTS

There are several key words or phrases that often help in converting words into algebraic statements.

1. **Addition:** Plus; add; more than; increased by; sum; total. **Ex:** "**4** more than a number" becomes $4 + n$; "a number increased by **3**" becomes $n + 3$.
2. **Subtraction:** Minus; subtract; decreased by; less than; difference. **Ex:** "**6** less than a number" becomes $n - 6$. It cannot be written $6 - n$, because the **6** is being taken away from the number, not the other way around; "a number decreased by **5**" becomes $n - 5$ and not $5 - n$; always consider which value is being subtracted.
3. **Multiplication:** Times; multiply; product; of (when used with a fraction); doubled; tripled. **Ex:** "$^2/_3$ of a number" becomes $(^2/_3)n$; "the product of 7 and a number" becomes $7n$.
4. **Division:** Divided by; divided into; quotient; a half (divide by **2**); a third (divide by **3**). **Ex:** "A number divided by **2**" becomes $n/2$; "the quotient of **8** and a number" becomes $8/n$.
5. **Inequality and equality symbols:**
   a. $>$ comes from "is greater than" or "is more than" and not "more than," which is addition.
   b. $<$ comes from "is less than" and not "less than," which is subtraction.
   c. $\geq$ comes from "is more than or equal to" or "is greater than or equal to."
   d. $\leq$ comes from "is less than or equal to."
   e. $\neq$ comes from "is not equal to."
   f. $=$ comes from "is equal to" or "equals."

ALGEBRAIC EQUATIONS

PROPERTIES

1. **Addition/Subtraction Property of Equality:** If $a = b$, then $a + c = b + c$ and $a - c = b - c$; that is, you can add or subtract any number or term to or from an equation as long as you do it on both sides of the equal sign.
2. **Multiplication/Division Property of Equality:** If $a = b$, then $ac = bc$ and $a/c = b/c$ (when $c \neq 0$); that is, you can multiply or divide by any number or term as long as you do it on both sides of the equal sign. Remember, do not divide by zero because it is undefined.
3. **Symmetric Property**: If $a = b$, then $b = a$; that is, two sides of an equation can be exchanged without changing any signs or terms in the equation. **Ex:** $3n + 7 = 8 - 2n$ becomes $8 - 2n = 3n + 7$.

SOLUTION METHODS

FIRST DEGREE, ONE VARIABLE

1. Solving an equation means you are finding the one numerical value that makes the equation true when it is put into the equation in place of the variable.
2. Using inverse operations is the best method for first-degree equations. Using inverse operations means you do the operation opposite to the one in the equation.
3. **One-step equations:** Equations having only one operation (+, –, ×, or ÷) with the variable require only one inverse operation. If the equation has addition, then you do subtraction; if subtraction, you do addition; if multiplication, you do division; if division, you do multiplication.

   Ex 1: $n + 7 = -3 \leftarrow$ 7 is added to n, so,
   $n + 7 - 7 = -3 - 7 \leftarrow$ Subtract **7** from **both** sides
   $n = -10 \leftarrow$ giving the solution of **–10**

   Ex 2: $\frac{a}{3} = 9 \leftarrow a$ is divided by **3**, so,
   $\frac{a}{3} \cdot 3 = 9 \cdot 3 \leftarrow$ Multiply by **3** on both sides
   $a = 27 \leftarrow$ giving the solution of **27**.
4. **Two-step equations**:
   a. Equations that have two operations connected to the variable require two operations that are the opposites of the ones that are in the equation. It is much easier to do **addition** or **subtraction** before doing multiplication or division. This is the opposite of the order of operations because you are doing inverse or opposite operations to solve the equations.

   Ex 1: $3x + 4 = -8 \leftarrow$ **4** was added, so,
   $3x + 4 - 4 = -8 - 4 \leftarrow$ subtract **4** on both sides
   $3x \div 3 = -12 \div 3 \leftarrow$ 3 was multiplied, so divide by **3** on both sides
   $x = -4 \leftarrow$ giving the solution of **–4**.

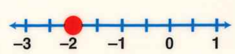
   Ex 2: $\frac{n}{2} - 7 = 3 \leftarrow$ 7 was subtracted, so,
   $\frac{n}{2} - 7 + 7 = 3 + 7 \leftarrow$ add **7** to both sides
   $\frac{n}{2} \cdot 2 = 10 \cdot 2 \leftarrow n$ is divided by **2**, so multiply by **2** on both sides
   $n = 20 \leftarrow$ giving the solution of **20**.

   b. If the equation has the variable on the right side of the equal sign, then it can be solved, leaving the variable on the right side, or it can be turned around by simply taking everything on each side of the equal sign and putting it on the opposite side without changing any signs or terms in any way (**symmetric property**).
5. **More than two-step equations**: Equations sometimes require simplifying each side of the equation separately before beginning to do inverse operations.

   Ex: $3(2n + 1) + 9 = 4n - 10 \leftarrow$ distribute **3**
   $6n + 3 + 9 = 4n - 10 \leftarrow$ add like terms
   $6n + 12 = 4n - 10 \leftarrow$ now begin inverse operations
   $6n + 12 - 4n = 4n - 10 - 4n \leftarrow$ subtract $4n$ on both sides
   $2n + 12 - 12 = -10 - 12 \leftarrow$ subtract **12** on both sides
   $2n \div 2 = -22 \div 2 \leftarrow$ divide by **2** on both sides
   $n = -11 \leftarrow$ giving the solution of **–11**.
6. **Proportions:** Equations in which both sides of the equal sign are fractions. The cross-multiplication rule can be used to solve such equations.
   The rule is that if $\frac{a}{c} = \frac{b}{d}$, then $ad = bc$.

   Ex 1: $\frac{5}{x} = \frac{3}{7}$
   $5 \cdot 7 = 3 \cdot x \leftarrow$ cross multiply
   $35 \div 3 = 3x \div 3 \leftarrow$ inverse operation, divide by **3**
   $11\frac{2}{3} = x \leftarrow$ solution

   Ex 2: $\frac{4}{5} = \frac{3}{(x+2)}$
   $4(x + 2) = 5 \cdot 3 \leftarrow$ cross multiply
   $4x + 8 = 15 \leftarrow$ distribute the 4
   $4x + 8 - 8 = 15 - 8 \leftarrow$ inverse operation; **– 8**
   $4x \div 4 = 7 \div 4 \leftarrow$ inverse operation; divide by **4**
   $x = 1.75 \leftarrow$ solution
7. **Graphing solutions:** Since equations have only one solution, the graphs of their solutions are simply a solid dot on the number on the real number line. **Ex:** If you solved the equation $4k-7 = -15$ and found the answer $k = -2$, then you would draw a real number line and put a solid dot on the line above **–2**, such as above right.



ALGEBRAIC INEQUALITIES

Algebraic inequalities are statements that do not have an equal sign but rather one of these symbols: $>, <, \geq, \leq,$ or $\neq$.

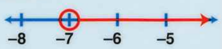
PROPERTIES

1. **Addition/Subtraction Property of Inequality:** If $a > b$, then $a + c > b + c$ and $a - c > b - c$. Also, if $a < b$, then $a + c < b + c$ and $a - c < b - c$. This means that you can add or subtract any number or term to or from both sides of the inequality.
2. **Multiplication/Division Property of Inequality:** If $a > b$, then $ac > bc$ and $a/c > b/c$ only if c is a positive number. If c is a negative number, then $a > b$ becomes $ac < bc$ or $a/c < b/c$. (Notice that if $\mathbf{8 > 5}$ and you multiply each side by -2, then you get $\mathbf{-16 > -10}$, which is false, but if you turn the symbol around, getting $\mathbf{-16 < -10}$, it becomes true again.) **Caution:** Turn the symbol around only when you multiply or divide by a negative number.

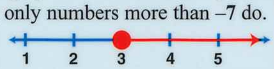
SOLUTION METHODS

FIRST DEGREE, ONE VARIABLE

1. Solving inequalities is exactly the same as solving equations, as discussed on *page 2*, with only one exception. The exception is when you multiply or divide by a negative number, the inequality symbol turns around to keep the inequality **true**, so you will get a true solution. The symbol does not turn around when you are adding or subtracting any terms or numbers or when you are multiplying or dividing by a positive number.
   Ex: $3(x + 2) > -15$
   $3x + 6 > -15 \leftarrow$ distribute the **3**
   $3x + 6 - 6 > -15 - 6 \leftarrow$ inverse operation (-6)
   $3x \div 3 > -21 \div 3 \leftarrow$ inverse operation $(\div 3)$
   $x > -7 \leftarrow$ solution
   NOTE: The $>$ symbol did NOT turn around because the division was by $+3$, not -21.
2. **Graphing solutions:** Inequalities have many solutions or answers, so the graphs of the solutions look very different from the graphs of equations.
   a. Graphs of equations usually have only one solid dot, but the graphs of inequalities have either solid dots with rays or open dots with rays.

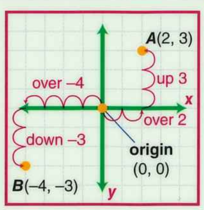


   Ex: If the solution to an inequality is $x > -7$, the graph above is with an open dot because -7 does NOT make the inequality true, only numbers more than -7 do.
   b. The solid dot shows that the number is part of the answer, but an open dot shows that the number is not part of the answer but only a beginning point. **Ex:** If the solution is $n \geq 3$, the graph above right shows a solid dot.



COORDINATE PLANE

POINTS

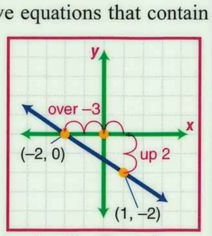


1. The coordinate plane is a grid with an **x-axis** and a **y-axis**.
2. Every point on a plane can be named using an ordered pair.
3. An **ordered pair** is two numbers separated by a comma and enclosed by parentheses (x, y). The first number is the x number and the second number is the y number.
   Ex: $(3, -5)$, where $x = 3$ and $y = -5$.
4. The point where the **x-axis** and the **y-axis** intersect or cross is called the **origin** and has the ordered pair **(0, 0)**.
5. The x number in the ordered pair tells you how far to go to the right (if positive) or to the left (if negative) from the origin **(0, 0)**.
6. The y number in the ordered pair tells you how far to go up (if positive) or down (if negative), either from the origin or from the last location found by using the x number.

LINES & EQUATIONS

1. The **coordinate plane** and **ordered pairs** are used to name all of the points on a plane.
2. When the points form a line, a special equation can be written to represent all of the points on the line.
3. Since points are named using ordered pairs with x numbers and y numbers in them, equations of lines, called linear equations, are written with the variables x and/or y in them. **Ex:** $2x + y = 5$; $y = x - 6$; $x = -2$; $y = 5$.
4. Lines that cross both the **x-axis** and the **y-axis** have equations that contain **both** the variables x and y.
5. Lines that cross the **x-axis** and do not cross the **y-axis** have equations that contain **only** the variable x and not the variable y.
6. Lines that cross the **y-axis** and do not cross the **x-axis** have equations that contain **only** the variable y and not the variable x.

SLOPE OF A LINE



1. Every line has a **slope**, except vertical lines (have no slope). The slope can be thought of as a kind of slant to the line.
2. Slope is found by **comparing** the positions of any two points on the line.
3. Slope is $(y_1 - y_2)/(x_1 - x_2)$. It is also described as the **(rise)/(run)** or **(the change in y)/(the change in x)**. **Ex:** On the graph at right, the slope of line "l_1" can be found using the formula:

$$\textbf{slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{0-(-2)}{-2-1} = \frac{2}{-3} = -\frac{2}{3} \text{ or } \textbf{slope} = \frac{\textbf{rise}}{\textbf{run}} = \frac{\textbf{up 2}}{\textbf{over } -3} = \frac{2}{-3} = -\frac{2}{3}.$$

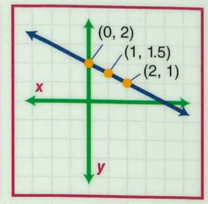
Notice that the values from the same point are aligned vertically, and the formula $\dfrac{y_2 - y_1}{x_2 - x_1}$ also works.

GRAPHING LINES

There are many ways to graph a linear equation.

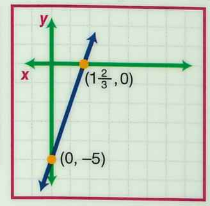
1. Pick any number to be the value of the x variable. Put it into the equation for the x and then solve the equation for the y. This gives **one ordered pair** (x, y), with the number you picked followed by the number you found when you solved the equation. You should pick at least **three** different values for x and solve, giving **3** points on the line. If the **3** points **don't** form a line, a mistake has been made on at least one of the equation solutions.

x	$x + 2y = 4$	y
0	$0 + 2y = 4$ $y = 2$	2
1	$1 + 2y = 4$ $y = 1.5$	1.5
2	$2 + 2y = 4$ $y = 1$	1

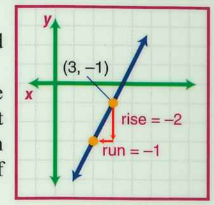


   Ex: The linear equation $x + 2y = 4$ can be put into a chart like the one above.
2. Find the points where the line crosses the **x-axis** (called the **x-intercept**) and the **y-axis** (called the **y-intercept**).
   a. This can be done by putting a zero into the equation for the x variable and solving for the y. This gives the point where the line crosses the **y-axis** because all points on the **y-axis** have x numbers of zero.

x	$3x - y = 5$	y
0	$3 \cdot 0 - y = 5$ $y = -5$	-5
$1\frac{2}{3}$	$3x - 0 = 5$ $x = 1\frac{2}{3}$	0



   b. Next, put a zero into the equation for the y number and solve for the x. This gives the point where the line crosses the **x-axis** because all points on the **x-axis** have y numbers of zero.
   Ex: The linear equation $\mathbf{3x - y = 5}$ could be put into a chart like the one above.
   c. Find one point on the line by:
      i. Putting a number into the equation for the x and solving for the y.
      ii. Next, use the slope of the line. The **slope** can be found in the equation. Look at the coefficient (number in front) of the x variable, change the sign of this number and divide it by the coefficient of the y variable. This is the slope of the line.



      iii. Then, graph the point you found and count the slope from that point using **(rise)/(run)**. **Ex:** The linear equation $\mathbf{2x - y = 7}$ goes through the point $\mathbf{(3, -1)}$. The slope is $\mathbf{-2/-1}$ because you change the sign of the number in front of the x variable and divide it by the coefficient of the y variable, which is $\mathbf{-1}$. Graph these values as above right. The slope-intercept form, $y = mx + b$, can also be used, where m is the slope and b is the y-intercept.

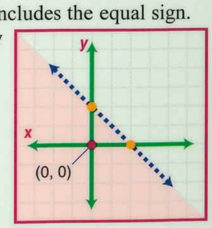
GRAPHING INEQUALITIES

On the coordinate plane, linear inequalities are line graphs with a shaded region included, either above or below the line.

1. Graph the **line** (even if the inequality does not include the equal sign, you must graph the corresponding equality).
2. Pick a point **above** the line.
3. Put the number values for x and for y into the inequality to see if they make the inequality **true**.
4. If the point makes the inequality true, shade that side of the line.
5. If the point makes the inequality false, shade the other side of the line.
6. The actual line is drawn as a **solid line** if the inequality includes the equal sign.
7. The actual line is drawn as a **dashed line** if the inequality does not include the equal sign. **Ex:** Graph $x + y < 2$.

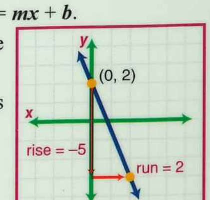
x	$x + y = 2$	y
0	$0 + y = 2$ $y = 2$	2
2	$x + 0 = 2$ $x = 2$	0

Test **(0, 0)** in
$x + y < 2$
$\mathbf{0 + 0 < 2}$
is true, so shade that side of the line.



FINDING LINEAR EQUATIONS

1. Some linear equations can be found by observing the relationship between the x numbers and the y numbers. **Ex:** A line with the points $(3, 2)$, $(5, 4)$, $(-2, -3)$, and $(-5, -6)$ has the equation $y = x - 1$, because every y number is one less than the x number.
2. Some linear equations require methods other than simple observation, such as the slope-intercept form of a linear equation (see #4 below).
3. The **standard form** of linear equations is $ax + by = c$, where a, b, and c are integers.
4. **The slope-intercept form of a linear equation** is $y = mx + b$, where the m represents the slope and the b represents the **y-intercept** of the line. One way to find the equation of a line is to find the **y-intercept** (where the line crosses the **y-axis**) and the slope, then put them into the slope-intercept form of a linear equation.
   a. Find the **y-intercept**, then use it to replace the b in $y = mx + b$.
   b. Next, find the slope of the line; use the slope to replace the m in $y = mx + b$.
   c. The result is the equation of the line with the number values in place of the m and the b in the form $y = mx + b$.
   d. **Ex:** $b = 2$ **and** $m = \dfrac{-5}{2}$. The equation: $y = -\dfrac{5}{2}x + 2$.
   Or, in standard form: $\mathbf{5x + 2y = 4}$.



QuickStudy

GEOMETRY

PLANE GEOMETRY

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5. **Definitions**
   a. A **line segment** is two points on a line (the endpoints) and all of the points between them. The notation for a line segment is the two endpoints (in capital letters) with a bar over them. **Ex:** Line segment PQ, written $\overline{PQ}$, is shaded in blue on the line 

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   b. A **midpoint** is the point in the center of a line segment. It separates a line segment into two equal parts.
   c. A **ray** is one point on a line (the endpoint) and all of the points on the line continuing on from the endpoint and going in one direction forever. The **notation** for a ray is the endpoint followed by any other point on the ray, with a ray pointing always to the right, drawn on top of the two capital letters. **Ex:** Ray AB, written $\overrightarrow{AB}$, is shaded in blue on the line $\overleftrightarrow{AB}$.


   d. **Intersecting lines** cross or touch in exactly one point. They are always coplanar (in the same plane).
   e. **Perpendicular lines** intersect or cross, forming 90-degree angles at the point of intersection.
   f. **Parallel lines** go in the same direction and never touch. They are always coplanar (in the same plane).
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   h. An **angle** is the union of two rays with a common endpoint. The common endpoint of the two rays becomes the **vertex** of the angle. The two rays become the sides of the angle. Angles are measured with protractors and in degrees.
      i. **Acute angles** have measures **less than 90** degrees.
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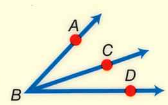
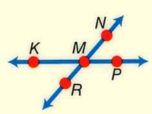
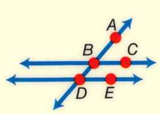
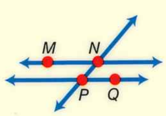


   l. **Vertical angles** are angles that have the same vertex and whose sides form lines. **Ex:** $\measuredangle KMN$ and $\measuredangle RMP$ are vertical angles. $\measuredangle KMR$ and $\measuredangle NMP$ are also **vertical** angles.


   m. **Corresponding angles** of three intersecting lines are two angles with one angle having its vertex and one side on the side of the other angle. The other two sides of both angles go on the same side of the first sides. **EX:** $\measuredangle ABC$ and $\measuredangle BDE$ are corresponding angles.


   n. **Alternate interior angles** are two angles that have one side of each angle overlapping and forming a line while the other sides of the angles go in opposite directions from different vertices (plural of vertex). **Ex:** $\measuredangle MNP$ and $\measuredangle NPQ$ are alternate interior angles.


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CIRCLES

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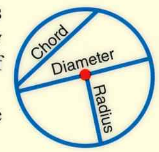
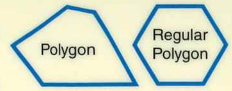


POLYGONS



1. **Polygons** are closed plane figures whose sides are line segments joined at the endpoints.
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   a. The sum of the angles of a triangle is **180°**.
   b. The sum of the lengths of any 2 sides of a triangle is **greater** than the length of the third side.
   c. Triangles can be **classified** in two ways, by their angles and by their side lengths.

BY ANGLES	
Right	**one 90° angle**
Obtuse	**one angle > 90°**
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BY SIDES	
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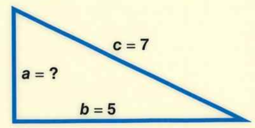
Isosceles	**2 or more sides equal**
Equilateral	**all sides equal**

THE PYTHAGOREAN THEOREM

The Pythagorean Theorem can be used to find the length of one side of any right triangle when given the length of the other two sides. It is usually written as $a^2 + b^2 = c^2$, where a and b are the lengths of the two legs and c is the length of the hypotenuse. **Ex:**



To find length of leg "*a*"

$$a^2 + b^2 = c^2$$
$$a^2 + (5)^2 = (7)^2$$
$$a^2 + 25 = 49$$
$$a^2 = 49 - 25$$
$$a = \sqrt{24}$$
$$a = 4.899$$

TRIG FUNCTIONS

Trigonometric (trig) functions of an angle can be used to find the measures of the angles and the lengths of the sides of right triangles.

1. There are 6 trig functions that are ratios, but only 3 will be discussed here.
   a. **sine** of an angle = **sin** $A = \dfrac{\textbf{opposite leg}}{\textbf{hypotenuse}}$
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2. Some things to note:
   a. The A represents an **acute angle** in the right triangle.
   b. The leg of the right triangle is considered either the **opposite** leg or the **adjacent** leg changes, depending on which of the acute angles of the right triangle is being evaluated in the trig function.
   c. The **opposite leg** of a right triangle is the leg that **does not** touch the vertex of the angle named in the trig function.
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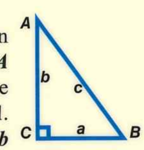
   e. **Ex:** When evaluating the trig functions for angle A in this right triangle, leg a is the opposite leg for angle A because it does not touch point A; however, leg b is the adjacent leg for angle A because it does touch point A. The hypotenuse is side c. In the same right triangle, leg b is the opposite leg for angle B because it does not touch point B; however, leg a is the adjacent leg for angle B because it does touch point B.


   f. **NOTICE**: The opposite leg for angle A is also the adjacent leg for angle B, and the adjacent leg for angle A is also the opposite leg for angle B.
3. Since trig functions are ratios (fractions), they are often converted to **decimal numbers**.
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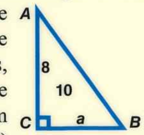

$$a^2 + b^2 = c^2$$
$$a^2 + (8)^2 = (10)^2$$
$$a^2 + 64 = 100$$
$$a^2 = 36$$
$$a = \sqrt{36}$$
$$a = 6$$

      ii. Once the three side lengths are found (not necessary, but it is easier), use the trig functions to find the degree measure of one acute angle.

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   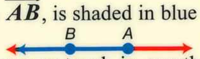

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   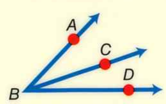

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   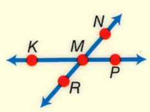

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   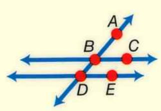

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   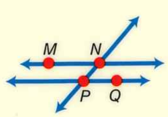

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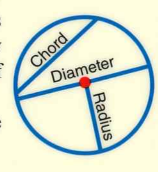

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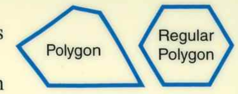

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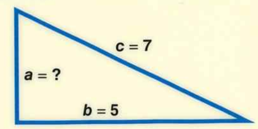

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   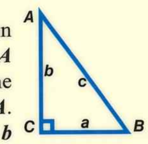

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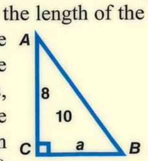
      i. To find the 2 acute angle measures when given 2 sides of a right triangle, it is easiest to find the length of the third side first.
      Ex: In the right triangle (at right), if you know the length of any two sides, then you may use the Pythagorean Theorem (**leg² + leg² = hypotenuse²**) to find the length of the third side.
      

      $$a^2 + b^2 = c^2$$
      $$a^2 + (8)^2 = (10)^2$$
      $$a^2 + 64 = 100$$
      $$a^2 = 36$$
      $$a = \sqrt{36}$$
      $$a = 6$$
      ii. Once the three side lengths are found (not necessary, but it is easier), use the trig functions to find the degree measure of one acute angle.

SCIENTIFIC NOTATION

A form of a decimal number where the decimal point is always behind exactly one non-zero digit and the number is multiplied by a power of ten.

Ex: 4.87×10^8; 3.981×10^{-6}.

1. It is a method for representing **very large** or **very small** numbers without writing a lot of digits. **Ex:** **243,700,000,000,000** would be written as **2.437×10^{14}**; **.000000982** would be written as **9.82×10^{-7}**.
2. A **positive** or **zero** exponent on the **10** means the number value is more than or equal to one. A **negative** exponent on the **10** means the number value is less than one. **Ex:** 5.29×10^{-10} = .000000000529 and 5.29×10^{14} = 529,000,000,000,000.
3. Operations with very large or very small numbers can be completed using the scientific notation form of the numbers, especially with calculators.

ALGEBRA CONCEPTS

DEFINITIONS

1. A **variable** is a letter that represents a number.
2. A **coefficient** is a number that is multiplied by the variable. It is found in front of a variable, but the multiplication sign is not written. If the coefficient is one, the one is not written. **Ex:** $5n = 5 \times n$; if n were **3**, then $5n$ would equal 5×3 or **15**.
3. A **term** is a mathematical expression involving multiplication or division. Terms are separated by an addition or subtraction sign. **Ex:** $7a$ is one term; $3k + 9$ is two terms; $4m^2 - 8m + 3$ is three terms.
4. **Like (or similar) terms** are terms that have the same variables and exponents, written in any order. The coefficients (numbers in front) **do not** have to be the same. **Ex:** $4m$ and $9m$ are like terms; $5a^2c$ and $-7a^2c$ are like terms; $3r^3$ and $-9r^2$ are not like terms because the exponents are not the same; $15z^4$ and $8t^4$ are not like terms because they do not have the same variables.

OPERATIONS & PROPERTIES

1. **Addition and Subtraction:** Only **like** (or similar) terms can be added or subtracted. Once it is determined that the terms are like terms, only the coefficients (numbers in front of the terms) are added or subtracted. **Ex:** $3n + 7n - 11n = 10n - 11n = 10n + (-11n) = -1n$ or simply $-n$; $14k^2 + 5n - 10k^2 - n = 4k^2 + 4n$.
2. **Multiplication:** **Any terms** can be multiplied. They **do not** have to be like terms. When multiplying terms, multiply the coefficients (numbers in front) and the matching variables. **Ex:** $(-3m^2n)(5m^4n) = -15m^6n^2$; remember, when multiplying, make sure the **bases** are the same, then add exponents.
3. **Division:** **Any terms** can be divided. They **do not** have to be like terms. Division is usually written in fraction form. When dividing terms, divide or reduce the coefficients (numbers in front) and the matching variables. Remember that, to divide with exponents, you must subtract the exponents once you match the same bases. **Ex:** $(30a^7c^2)/(-6a^4c^3d^2) = (-5a^3)/(cd^2)$ because **30** divided by **–6** is **–5**, a^7 divided by a^4 is a^3, c^3 divided by c^2 is c, and there is no other variable d to divide by the d^2, so it remains the same.
4. **Commutative Property:** $a + b = b + a$ and $a - b = a + (-b) = (-b) + a$; therefore, terms can be moved as long as you take the proper sign (negative or positive) with the term. **Ex:** $4p^2 + 8p^3 = 8p^3 + 4p^2$; $14c - 3f = (-3f) + 14c$.
5. **Associative Property:** $(a + b) + c = a + (b + c)$ and $(a - b) - c = a + (-b + -c)$; therefore, terms can be added in any order as long as all subtraction is first changed to addition. **Ex:** $(5j - 8j) - 12j = 5j + (-8j + -12j)$.
6. **Distributive Property:** $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$; therefore, if the terms inside the parentheses cannot be added or subtracted, multiply them BOTH or ALL by the value located in **front** of the parentheses. **Ex:** $3n(5n + 6) = 15n^2 + 18n$; $a^2c(5a^2 + 2ac - c^2) = 5a^4c + 2a^3c^2 - a^2c^3$.
7. **Double Negative Property:** $-(-a) = a$; therefore, if there is a negative of a negative, it becomes positive, just like a negative number times a negative number equals a positive number.

TRANSLATING

PUTTING WORDS INTO ALGEBRAIC STATEMENTS

There are several key words or phrases that often help in converting words into algebraic statements.

1. **Addition:** Plus; add; more than; increased by; sum; total. **Ex:** "4 more than a number" becomes $4 + n$; "a number increased by 3" becomes $n + 3$.
2. **Subtraction:** Minus; subtract; decreased by; less than; difference. **Ex:** "6 less than a number" becomes $n - 6$. It cannot be written $6 - n$, because the **6** is being taken away from the number, not the other way around; "a number decreased by 5" becomes $n - 5$ and not $5 - n$; always consider which value is being subtracted.
3. **Multiplication:** Times; multiply; product; of (when used with a fraction); doubled; tripled. **Ex:** "2/3 of a number" becomes (2/3)n; "the product of 7 and a number" becomes **7n**.
4. **Division:** Divided by; divided into; quotient; a half (divide by **2**); a third (divide by **3**). **Ex:** "A number divided by 2" becomes $n/2$; "the quotient of 8 and a number" becomes $8/n$.
5. **Inequality and equality symbols:**
   a. > comes from "is greater than" or "is more than" and not "more than," which is addition.
   b. < comes from "is less than" and not "less than," which is subtraction.
   c. ≥ comes from "is more than or equal to" or "is greater than or equal to."
   d. ≤ comes from "is less than or equal to."
   e. ≠ comes from "is not equal to."
   f. = comes from "is equal to" or "equals."

ALGEBRAIC EQUATIONS

PROPERTIES

1. **Addition/Subtraction Property of Equality:** If $a = b$, then $a + c = b + c$ and $a - c = b - c$; that is, you can add or subtract any number or term to or from an equation as long as you do it on both sides of the equal sign.
2. **Multiplication/Division Property of Equality:** If $a = b$, then $ac = bc$ and $a/c = b/c$ (when $c \neq 0$); that is, you can multiply or divide by any number or term as long as you do it on both sides of the equal sign. Remember, do not divide by zero because it is undefined.
3. **Symmetric Property:** If $a = b$, then $b = a$; that is, two sides of an equation can be exchanged without changing any signs or terms in the equation. **Ex:** $3n + 7 = 8 - 2n$ becomes $8 - 2n = 3n + 7$.

SOLUTION METHODS

FIRST DEGREE, ONE VARIABLE

1. Solving an equation means you are finding the one numerical value that makes the equation true when it is put into the equation in place of the variable.
2. Using inverse operations is the best method for first-degree equations. Using inverse operations means you do the operation opposite to the one in the equation.
3. **One-step equations:** Equations having only one operation (+, –, ×, or ÷) with the variable require only one inverse operation. If the equation has addition, then you do subtraction; if subtraction, you do addition; if multiplication, you do division; if division, you do multiplication.

   Ex 1: $n + 7 = -3 \leftarrow$ 7 is added to n, so,
   $n + 7 - 7 = -3 - 7 \leftarrow$ Subtract **7** from **both** sides
   $n = -10 \leftarrow$ giving the solution of **–10**

   Ex 2: $\frac{a}{3} = 9 \leftarrow a$ is divided by **3**, so,
   $\frac{a}{3} \cdot 3 = 9 \cdot 3 \leftarrow$ Multiply by **3** on both sides
   $a = 27 \leftarrow$ giving the solution of **27**.
4. **Two-step equations:**
   a. Equations that have two operations connected to the variable require two operations that are the opposites of the ones that are in the equation. It is much easier to do **addition** or **subtraction** before doing multiplication or division. This is the opposite of the order of operations because you are doing inverse or opposite operations to solve the equations.

   Ex 1: $3x + 4 = -8 \leftarrow$ **4** was added, so,
   $3x + 4 - 4 = -8 - 4 \leftarrow$ subtract **4** on both sides
   $3x \div 3 = -12 \div 3 \leftarrow$ 3 was multiplied, so divide by **3** on both sides
   $x = -4 \leftarrow$ giving the solution of **–4**.

   Ex 2: $\frac{n}{2} - 7 = 3 \leftarrow$ **7** was subtracted, so,
   $\frac{n}{2} - 7 + 7 = 3 + 7 \leftarrow$ add **7** to both sides
   $\frac{n}{2} \cdot 2 = 10 \cdot 2 \leftarrow n$ is divided by **2**, so multiply by **2** on both sides
   $n = 20 \leftarrow$ giving the solution of **20**.

   b. If the equation has the variable on the right side of the equal sign, then it can be solved, leaving the variable on the right side, or it can be turned around by simply taking everything on each side of the equal sign and putting it on the opposite side without changing any signs or terms in any way (**symmetric property**).
5. **More than two-step equations:** Equations sometimes require simplifying each side of the equation separately before beginning to do inverse operations.

   Ex: $3(2n + 1) + 9 = 4n - 10 \leftarrow$ distribute **3**
   $6n + 3 + 9 = 4n - 10 \leftarrow$ add like terms
   $6n + 12 = 4n - 10 \leftarrow$ now begin inverse operations
   $6n + 12 - 4n = 4n - 10 - 4n \leftarrow$ subtract **4n** on both sides
   $2n + 12 - 12 = -10 - 12 \leftarrow$ subtract **12** on both sides
   $2n \div 2 = -22 \div 2 \leftarrow$ divide by **2** on both sides
   $n = -11 \leftarrow$ giving the solution of **–11**.
6. **Proportions:** Equations in which both sides of the equal sign are fractions. The cross-multiplication rule can be used to solve such equations.

   The rule is that if $\frac{a}{c} = \frac{b}{d}$, then $ad = bc$.

   Ex 1: $\frac{5}{x} = \frac{3}{7}$
   $5 \cdot 7 = 3 \cdot x \leftarrow$ cross multiply
   $35 \div 3 = 3x \div 3 \leftarrow$ inverse operation, divide by 3
   $11\frac{2}{3} = x \leftarrow$ solution

   Ex 2: $\frac{4}{5} = \frac{3}{(x+2)}$
   $4(x + 2) = 5 \cdot 3 \leftarrow$ cross multiply
   $4x + 8 = 15 \leftarrow$ distribute the 4
   $4x + 8 - 8 = 15 - 8 \leftarrow$ inverse operation; **– 8**
   $4x \div 4 = 7 \div 4 \leftarrow$ inverse operation; divide by **4**
   $x = 1.75 \leftarrow$ solution
7. **Graphing solutions:** Since equations have only one solution, the graphs of their solutions are simply a solid dot on the number on the real number line. **Ex:** If you solved the equation $4k - 7 = -15$ and found the answer $k = -2$, then you would draw a real number line and put a solid dot on the line above **–2**, such as above right.

ALGEBRAIC INEQUALITIES

Algebraic inequalities are statements that do not have an equal sign but rather one of these symbols: >, <, ≥, ≤, or ≠.

PROPERTIES

1. **Addition/Subtraction Property of Inequality:** If $a > b$, then $a + c > b + c$ and $a - c > b - c$. Also, if $a < b$, then $a + c < b + c$ and $a - c < b - c$. This means that you can add or subtract any number or term to or from both sides of the inequality.
2. **Multiplication/Division Property of Inequality:** If $a > b$, then $ac > bc$ and $a/c > b/c$ only if c is a positive number. If c is a negative number, then $a > b$ becomes $ac < bc$ or $a/c < b/c$. (Notice that if **8 > 5** and you multiply each side by –2, then you get **–16 > –10**, which is false, but if you turn the symbol around, getting **–16 < –10**, it becomes true again.) **Caution:** Turn the symbol around only when you multiply or divide by a negative number.

SOLUTION METHODS

FIRST DEGREE, ONE VARIABLE

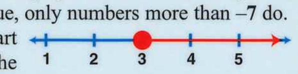
1. Solving inequalities is exactly the same as solving equations, as discussed on *page 2*, with only one exception. The exception is when you multiply or divide by a negative number, the inequality symbol turns around to keep the inequality **true**, so you will get a true solution. The symbol does not turn around when you are adding or subtracting any terms or numbers or when you are multiplying or dividing by a positive number.

   Ex: $3(x + 2) > -15$
   $3x + 6 > -15 \leftarrow$ distribute the **3**
   $3x + 6 - 6 > -15 - 6 \leftarrow$ inverse operation (-6)
   $3x \div 3 > -21 \div 3 \leftarrow$ inverse operation $(\div 3)$
   $x > -7 \leftarrow$ solution

   NOTE: The > symbol did NOT turn around because the division was by **+ 3**, not **– 21**.
2. **Graphing solutions:** Inequalities have many solutions or answers, so the graphs of the solutions look very different from the graphs of equations.
   a. Graphs of equations usually have only one solid dot, but the graphs of inequalities have either solid dots with rays or open dots with rays.

   Ex: If the solution to an inequality is $x > -7$, the graph above is with an open dot because –7 does NOT make the inequality true, only numbers more than –7 do.

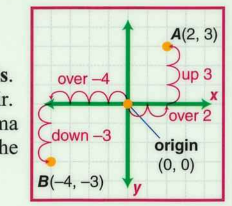
   b. The solid dot shows that the number is part of the answer, but an open dot shows that the number is not part of the answer but only a beginning point. **Ex:** If the solution is $n \geq 3$, the graph above right shows a solid dot.



COORDINATE PLANE

POINTS

1. The coordinate plane is a grid with an **x-axis** and a **y-axis**.
2. Every point on a plane can be named using an ordered pair.
3. An **ordered pair** is two numbers separated by a comma and enclosed by parentheses (x, y). The first number is the x number and the second number is the y number. **Ex:** $(3, -5)$, where $x = 3$ and $y = -5$.

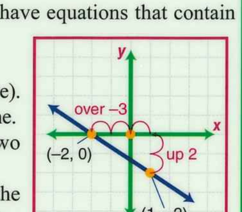


4. The point where the **x-axis** and the **y-axis** intersect or cross is called the **origin** and has the ordered pair **(0, 0)**.
5. The x number in the ordered pair tells you how far to go to the right (if positive) or to the left (if negative) from the origin **(0, 0)**.
6. The y number in the ordered pair tells you how far to go up (if positive) or down (if negative), either from the origin or from the last location found by using the x number.

LINES & EQUATIONS

1. The **coordinate plane** and **ordered pairs** are used to name all of the points on a plane.
2. When the points form a line, a special equation can be written to represent all of the points on the line.
3. Since points are named using ordered pairs with x numbers and y numbers in them, equations of lines, called linear equations, are written with the variables x and/or y in them. **Ex:** $2x + y = 5$; $y = x - 6$; $x = -2$; $y = 5$.
4. Lines that cross both the **x-axis** and the **y-axis** have equations that contain **both** the variables x and y.
5. Lines that cross the **x-axis** and do not cross the **y-axis** have equations that contain **only** the variable x and not the variable y.
6. Lines that cross the **y-axis** and do not cross the **x-axis** have equations that contain **only** the variable y and not the variable x.

SLOPE OF A LINE



1. Every line has a **slope**, except vertical lines (have no slope). The slope can be thought of as a kind of slant to the line.
2. Slope is found by **comparing** the positions of any two points on the line.
3. Slope is $(y_1 - y_2)/(x_1 - x_2)$. It is also described as the **(rise)/(run)** or **(the change in y)/(the change in x)**. **Ex:** On the graph at right, the slope of line "l_1" can be found using the formula:

$$\textbf{slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{0-(-2)}{-2-1} = \frac{2}{-3} = -\frac{2}{3} \text{ or } \textbf{slope} = \frac{\textbf{rise}}{\textbf{run}} = \frac{\textbf{up 2}}{\textbf{over } -3} = \frac{2}{-3} = -\frac{2}{3}.$$

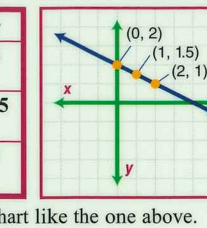
Notice that the values from the same point are aligned vertically, and the formula $\frac{y_2 - y_1}{x_2 - x_1}$ also works.

GRAPHING LINES

There are many ways to graph a linear equation.

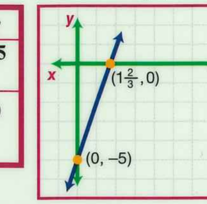
1. Pick any number to be the value of the x variable. Put it into the equation for the x and then solve the equation for the y. This gives **one ordered pair** (x, y), with the number you picked followed by the number you found when you solved the equation. You should pick at least **three** different values for x and solve, giving **3** points on the line. If the **3** points **don't** form a line, a mistake has been made on at least one of the equation solutions.

x	$x + 2y = 4$	y
0	$0 + 2y = 4$, $y = 2$	**2**
1	$1 + 2y = 4$, $y = 1.5$	**1.5**
2	$2 + 2y = 4$, $y = 1$	**1**



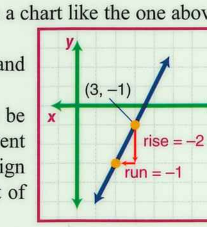
   Ex: The linear equation $x + 2y = 4$ can be put into a chart like the one above.
2. Find the points where the line crosses the **x-axis** (called the **x-intercept**) and the **y-axis** (called the **y-intercept**).
   a. This can be done by putting a zero into the equation for the x variable and solving for the y. This gives the point where the line crosses the **y-axis** because all points on the **y-axis** have x numbers of zero.

x	$3x - y = 5$	y
0	$3 \cdot 0 - y = 5$, $y = -5$	**–5**
$1\frac{2}{3}$	$3x - 0 = 5$, $x = 1\frac{2}{3}$	**0**



   b. Next, put a zero into the equation for the y number and solve for the x. This gives the point where the line crosses the **x-axis** because all points on the **x-axis** have y numbers of zero.

   Ex: The linear equation $3x - y = 5$ could be put into a chart like the one above.
   c. Find one point on the line by:
      i. Putting a number into the equation for the x and solving for the y.
      ii. Next, use the slope of the line. The **slope** can be found in the equation. Look at the coefficient (number in front) of the x variable, change the sign of this number and divide it by the coefficient of the y variable. This is the slope of the line.
      iii. Then, graph the point you found and count the slope from that point using **(rise)/(run)**. **Ex:** The linear equation $2x - y = 7$ goes through the point **(3, –1)**. The slope is **–2/–1** because you change the sign of the number in front of the x variable and divide it by the coefficient of the y variable, which is **–1**. Graph these values as above right. The slope-intercept form, $y = mx + b$, can also be used, where m is the slope and b is the y-intercept.



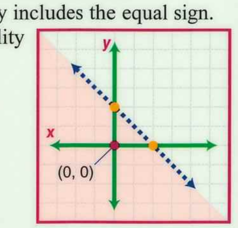
GRAPHING INEQUALITIES

On the coordinate plane, linear inequalities are line graphs with a shaded region included, either above or below the line.

1. Graph the **line** (even if the inequality does not include the equal sign, you must graph the corresponding equality).
2. Pick a point **above** the line.
3. Put the number values for x and for y into the inequality to see if they make the inequality **true**.
4. If the point makes the inequality true, shade that side of the line.
5. If the point makes the inequality false, shade the other side of the line.
6. The actual line is drawn as a **solid line** if the inequality includes the equal sign.
7. The actual line is drawn as a **dashed line** if the inequality does not include the equal sign. **Ex:** Graph $x + y < 2$.

x	$x + y = 2$	y
0	$0 + y = 2$, $y = 2$	**2**
2	$x + 0 = 2$, $x = 2$	**0**

Test **(0, 0)** in
$x + y < 2$
$0 + 0 < 2$
is true, so shade that side of the line.



FINDING LINEAR EQUATIONS

1. Some linear equations can be found by observing the relationship between the x numbers and the y numbers. **Ex:** A line with the points (3, 2), (5, 4), (–2, –3), and (–5, –6) has the equation $y = x - 1$, because every y number is one less than the x number.
2. Some linear equations require methods other than simple observation, such as the slope-intercept form of a linear equation (see #4 below).
3. The **standard form** of linear equations is $ax + by = c$, where a, b, and c are integers.
4. **The slope-intercept form of a linear equation** is $y = mx + b$, where the m represents the slope and the b represents the **y-intercept** of the line. One way to find the equation of a line is to find the **y-intercept** (where the line crosses the **y-axis**) and the slope, then put them into the slope-intercept form of a linear equation.
   a. Find the **y-intercept**, then use it to replace the b in $y = mx + b$.
   b. Next, find the slope of the line; use the slope to replace the m in $y = mx + b$.
   c. The result is the equation of the line with the number values in place of the m and the b in the form $y = mx + b$.
   d. **Ex:** $b = 2$ and $m = \frac{-5}{2}$. The equation: $y = -\frac{5}{2}x + 2$. Or, in standard form: $5x + 2y = 4$.